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The Fuzzy Future: Embracing the Potential of Fuzzy Functions

المستقبل الضبابي: استكشاف إمكانيات الوظائف الضبابية

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Abstract

The majority of the tools we have previously utilized for formal modeling, reasoning, and computing exhibit clarity, predictability, and precision. When we say "crisp," we are referring to a response that is limited to either a clear yes or no, without any ambiguity or shades of gray. In classical binary logic, a proposition can be either true or false, but not both simultaneously. In the field of set theory, an element can be classified as either a member of a set or not. In the field of optimization, a solution can be classified as either feasible or infeasible. Precision refers to the accurate representation of the parameters of a model, reflecting either our perception of the phenomenon being modeled or the functioning of the actual system being represented. Precision often entails a clear and unambiguous model.

This study primarily focuses on three main topics. Firstly, it provides a comprehensive treatment of fuzzy sets of type n, where n is an integer greater than or equal to 1. This includes an example, mathematical discussions, and real-life interpretations of the underlying mathematical concepts. Secondly, it explores the potentials and connections between fuzzy logic and probability logic, which have not been previously discussed in a single document. Lastly, it examines the representation of random and fuzzy uncertainties and ambiguities that arise in data-driven systems.

Keywords: Fuzzy sets, fuzzy logic, probability, and members of elements

الملخص

تُظهر غالبية الأدوات التي استخدمناها سابقًا للنمذجة الرسمية والاستدلال والحوسبة الوضوح والقدرة على التنبؤ والدقة. عندما نقول "واضح"، فإننا نشير إلى إجابة تقتصر على الإجابة بنعم أو لا واضحة، دون أي غموض أو ظلال رمادية. في المنطق الثنائي الكلاسيكي، يمكن أن يكون الاقتراح إما صحيحًا أو خاطئًا، ولكن ليس كلاهما في آنٍ واحد. في مجال نظرية المجموعات، يمكن تصنيف عنصر ما على أنه إما عضو في مجموعة أو لا. في مجال التحسين، يمكن تصنيف الحل إما ممكن أو غير ممكن. تشير الدقة إلى التمثيل الدقيق لمعلمات النموذج، مما يعكس إما إدراكنا للظاهرة التي يتم تمثيلها أو أداء النظام الفعلي الذي يتم تمثيله. وغالباً ما تستلزم الدقة نموذجاً واضحاً لا لبس فيه.

تركز هذه الدراسة في المقام الأول على ثلاثة مواضيع رئيسية. أولاً، تقدم معالجة شاملة للمجموعات الضبابية من النوع n، حيث n عدد صحيح أكبر من أو يساوي 1. يتضمن ذلك مثالاً ومناقشات رياضية وتفسيرات واقعية للمفاهيم الرياضية الأساسية. ثانياً، يستكشف الكتاب الإمكانات والروابط بين المنطق الضبابي ومنطق الاحتمالات، والتي لم تتم مناقشتها من قبل في وثيقة واحدة. وأخيراً، يبحث في تمثيل أوجه عدم اليقين والغموض العشوائية والضبابية التي تنشأ في الأنظمة التي تعتمد على البيانات.

الكلمات المفتاحية: المجموعات الضبابية، والمنطق الضبابي، والاحتمالية، وأعضاء العناصر

Introduction

Knowledge-based systems are computational algorithms designed to emulate human intelligence with the purpose of making judgments and providing recommendations. Knowledge-based systems encompass the renowned Expert System. Although these systems are commonly used, they are unable to accurately represent the uncertainties and lack of clarity in real-world problems. Expert system researchers mostly disregarded fuzzy logic following its debut by Lotfi Zadeh in 1965. "The concept of a linguistic variable was not well received initially," stated Zadeh, "mainly due to the conflict between my support for using words in systems and decision analysis and the long-standing tradition of valuing numbers and disregarding words." The development of fuzzy logic aimed to reveal the concealed aspects of conventional reasoning. The most effective approach to replicate human mind is by employing this particular form of reasoning. Fuzzy logic, which has existed for four decades, is currently experiencing a surge in popularity within the realm of artificial intelligence. Expert systems are utilizing it to address the vagueness and unpredictability of real-world problems. A Fuzzy Expert System is an expert system that utilizes a collection of fuzzy sets and rules to facilitate reasoning.

From an epistemological standpoint, this stance appears to presuppose that the concept will be constructed incrementally. One issue is that each level of theoretical development requires the addition of more formal and informal components due to the weak premises. As an example, a set of natural axioms (such as continuity, commutativity, monotonicity, etc.) is typically the starting point for a suitable mathematical formulation of the basic set-theory operators, including conjunction, disjunction, and complement. Despite their soundness, these axioms are common sense-based and apply exclusively to the set of things that need definition. As the formal apparatus increases, the efficacy of the procedure naturally decreases.

This is particularly evident in the domain of fuzzy logic, which is intimately connected. Paris (1994) provides a collection of fundamental principles for negation, conjunction, and disjunction. However, he does not get into the specifics of an implication-like connective because determining the appropriate axioms for this function is somewhat more ambiguous. Undoubtedly, the problem does not improve when a fuzzy-logical consequence relation is implemented.

The primary thesis of this study posits that fuzzy sets can be redefined inside an appropriate logical framework as a derived concept. To be more precise, fuzzy sets can be derived from a logical "deep structure" that governs their behavior "on the surface." Despite the complexity of the modal probabilistic framework required, it yields a more lucid explanation of our epistemological foundations. Furthermore, the methodology employed appears to maintain the integrity of the initial framework. It can be demonstrated that the suggested framework has the capability to encode many established fuzzy approaches.

Research Aims

The objective of this Special Issue is to delve further into the recent advancements in fuzzy set theory and its expansion to encompass applications in various domains of mathematics and engineering, including group theory, ring theory, statistics, topological spaces, graph theory, decision making, and other fields of applied science.

Research significance

We examine the necessary conditions for ensuring that all components of the fuzzy function are clearly described, with a detailed exploration of the types and interconnections of fuzzy functions and sets. Moreover, when two fuzzy functions are transformed into "fuzzified" versions of the same ordinary function, we investigate the characteristics and connection between the associated fuzzy functions.

Research Methodology

In this study, the researcher utilized a qualitative methodology to collect data from published research articles and periodicals.

Definition of Fuzzy logic

Fuzzy logic, unlike binary truth, allows for the consideration of multiple truth values rather of being limited to only 1 or 0.

Choosing between "Yes" or "No" is not always feasible in real-life situations. This is due to the possibility of being placed in a situation where you lack sufficient information to make a well-informed decision. Alternatively, it is possible that you are experiencing some level of confusion. It is unlikely that you will give an immediate affirmative or negative response to a query about your availability on a particular date next month due to uncertainty surrounding your intentions. I understand, it is challenging. Fuzzy logic is an artificial intelligence technique that enables computers to effectively handle unclear incoming data.

A machine learning framework or artificial intelligence system may utilize fuzzy logic as a method for decision-making. In simple terms, it refers to the procedure of evaluating the values of real-world variables inside the range of 0 and 1. Fuzzy logic is used to represent real numbers inside the range between 0 and 1.

The adjective "fuzzy" is employed to characterize something that lacks clarity or precision. The computer may lack the capability to ascertain the veracity or falsity of a particular situation. Boolean logic assigns the value of 1 to represent "True" and the value of 0 to represent "False." On the contrary, a fuzzy logic approach considers all the ambiguous aspects of a problem, where there might exist more than simply two potential solutions (True and False).

Professor Lotfi Zadeh, based at the University of California at Berkeley, pioneered the concept of fuzzy logic during the 1960s. He believed that conventional computer logic was inadequate for handling ambiguous or erroneous data. A computer, like to a human, has the ability to contemplate a range of options other than simply True and False. These options include: affirmative, potentially affirmative, uncertain, potentially negative, and negative.

Fuzzy logic, derived from human decision-making processes, proves valuable in representing intricate problems involving inaccurate or insufficient data. Fuzzy logic programs are more readily implemented than their logical and object-oriented counterparts due to their familiarity with everyday language. Furthermore, due to the reduced number of instructions, the operational memory requirements are also decreased.

A set is a clear idea in classical logic: a mathematical object (such as a number, partition, matrix, variable, etc.) either "belongs to" the set, in which case its degree of membership to the set is 1, or "does not belong to" the set, in which case its degree of membership to the set is 0. Therefore, a crisp set X may be expressed as a collection of mathematical objects, e.g.,

$X= \{X_1, X_2, \ldots, X_n\}$

Similarly, a proposition in classical logic is either "true" (may be quantified by a crisp value 1) or "false" (may be quantified by a crisp value 0).

In fuzzy logic, sets are fuzzy concepts. Our main focus is on the general case of type-n fuzzy sets, with n=1,2,3,.... To motivate this, we start with an example.

Let's say that, based on the fact that Felix is 27 years old, we would like to know whether or not Felix is a young person. Given that 27 is a quantitative concept and that young is a qualitative one, it is necessary to define young in order to relate the quantitative data on Felix's age to the quantified definition of young.

The graph in Figure 1 can be used to define young; it corresponds to a definition that divides people into two clear categories, young and not young, i.e., people under the age of 40 belong to the young set, and everyone else is not young. So, in terms of quantity, "Felix has a membership degree of 1 in the set young," while in terms of quality, "Felix is unquestionably young."

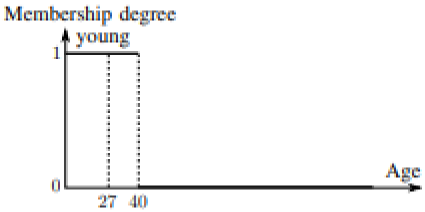

Figure 1: Using crisp sets for quantifying young.

Instead, young can be characterized using the graph in Figure 2, where ages fluctuate along a spectrum rather than being categorically young or obviously old, i.e., the degree of belonging to the set young varies in [0,1] rather than [0,1]. Thus, "Felix has a membership degree of 0.9 and is a member of the set young." In terms of quality, "Felix is largely young." After that, Young is quantified with a type 1 fuzzy set.

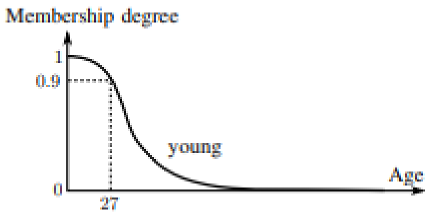

Figure 2: Using type-1 fuzzy sets for quantifying young.

Furthermore, let us assume that there is a lack of specific understanding on the exact definition of the boundary of the curve that represents youth. Figure 3 illustrates that the degree of blackness indicates the level of confidence we have in the placement of any point on the dividing curve within the limits of the given two-dimensional plane. By observing the vertical axis, one may determine that Felix's primary membership degree in the set "young" falls between the range of [0.57, 0.98]. Alternatively, it can be said that Felix is predominantly young. Imagine a third dimension that quantifies the intensity of the color black using real numbers ranging from 0 to 1, where a value of 1 represents pure black and a value of 0 represents pure white. (refer to figure 4). This dimension represents the secondary level of membership. Felix is a youthful individual, possessing a primary membership degree that falls within the range of [0.57, 0.98] to [0, 1], and a

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secondary membership degree that ranges from [0, 1]. "For example, Felix's age corresponds to a secondary membership degree of 0.83, given a primary membership degree of 0.88." In this situation, the concept of "young" has been measured using a type-2 fuzzy set.

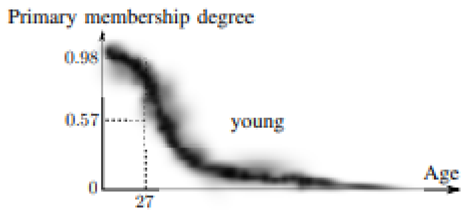

Figure 3: Using type–2 fuzzy sets for quantifying young.

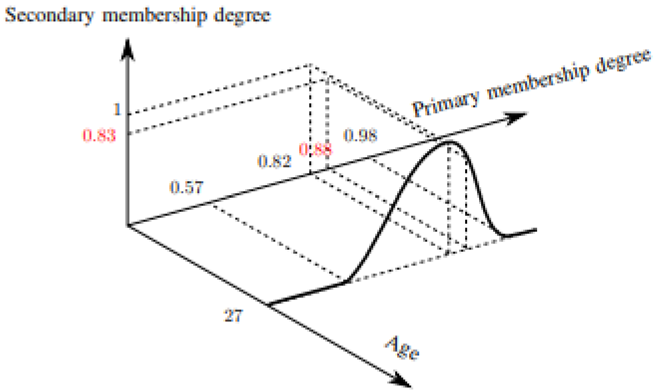

Figure 4: 3D representation of the type–2 membership function for quantifying young, represented for the specific age of 27.

Where is Fuzzy Logic Utilized?

There are extremely few places that are directly accessible. The Sony PalmTop is the sole example of a pure fuzzy logic application that I am aware of. It utilized a fuzzy logic decision tree method to accurately recognize handwritten Kanji characters using a computer lightpen. The primary application of fuzzy logic, as far as I am aware, is as the fundamental logical framework for fuzzy expert systems.

Logic Operations

Fuzzy logic is commonly employed to simulate human cognitive processes in artificial environments. It prioritizes credible thinking, which aligns more closely with the functioning of the real world. It is designed to handle ambiguity and excels at projecting outcomes. The steps involved in utilizing fuzzy logic are as follows:

Identify the object of management, the suitable course of action, any possible system issues, and the method of utilizing the system.

Establish the correlation between the input, the output, any deviations or mistakes, and the anomalies: In order for fuzzy logic to be effective, users need to initially determine the most significant inputs, as well as specify a suitable error rate and a minimal number of input variables.

To begin designing an if/then rule, the initial stage involves breaking down the control problem into a sequence of "IF A AND B THEN C" rules, utilizing the rule-based architecture of fuzzy logic. These rules will establish the intended course of action for specific inputs. The intricacy of a rule is directly related to the quantity of inputs and the number of variables linked to those inputs.

Please provide the code for the function. Create a membership function for fuzzy logic that precisely defines the relative significance of input and output data.

Please provide a detailed explanation of a standard business cycle: Create a systematic procedure for fuzzy logic both prior to and after to its implementation in hardware or software.

Demonstrate the effectiveness of your configurations: You can optimize membership functions and rules by doing system testing and evaluating the resulting outcomes. Continuously engage in a process of trial and refinement until you successfully achieve your objectives.

Probabilistic Logic

This section presents a comparison between fuzzy logic and probabilistic logic. In mathematical logic, a proposition can be likened to a mathematical object that can be classified as either a true or false statement, utilizing the aforementioned example of crisp or fuzzy sets. The explanations in this section are concise and rely exclusively on the concept of sets. The symbols T (representing true) and F (representing false) are employed to denote the sets that constitute the potential world, W, of an event E. This is done to illustrate the connection between sets and propositions.

An important distinction between fuzzy logic and probabilistic logic lies in the fact that F and T are fuzzy sets in fuzzy logic, but they are crisp sets in probabilistic logic. Consequently, there is confusion regarding the boundaries of the F and T values in fuzzy logic. Consequently, there can be instances where these two sets intersect; for instance, we might have T∏F≠Φ (refer to Figure 5), but based on probabilistic reasoning, we must also have T∏F=Φ (refer to Figure 6).

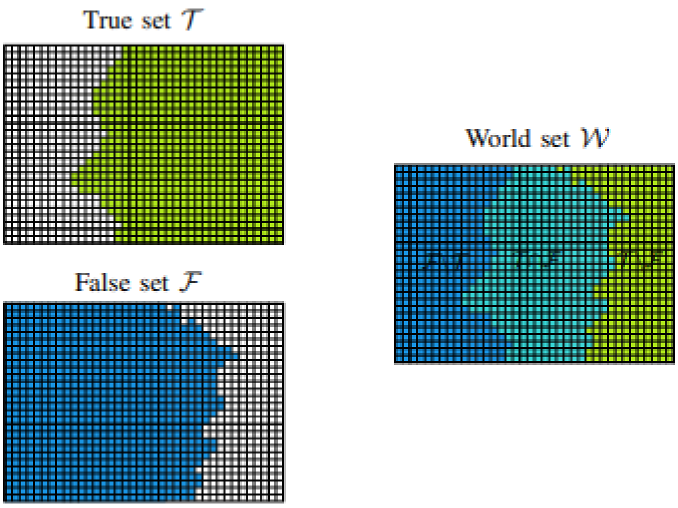

Figure 5: The sets of true T and false F propositions in fuzzy logic may have an overlap, i.e., T ∩F≠∅. Moreover, some mathematical objects that belong to F\T or T \F may not necessarily have a membership degree of 1.

A mathematical object z can correspond to any occurrence of the recurring event E. In the context of probabilistic logic, if event E occurs in a total of II1 experiments and in a separate set of II2 experiments, resulting in z being true for F and z being true for T, respectively, then the sum of II1 and II2 is equal to 100. The probability of belonging to F or T is determined by the normalized values π1 and π2, which correspond to the natural values II1 and II2, respectively. If z converges to F in experiment M1 and z reaches T in experiment M2, then the sum of M1 and M2 does not necessarily equal 100 experiments of fuzzy logic (it may exceed 100).

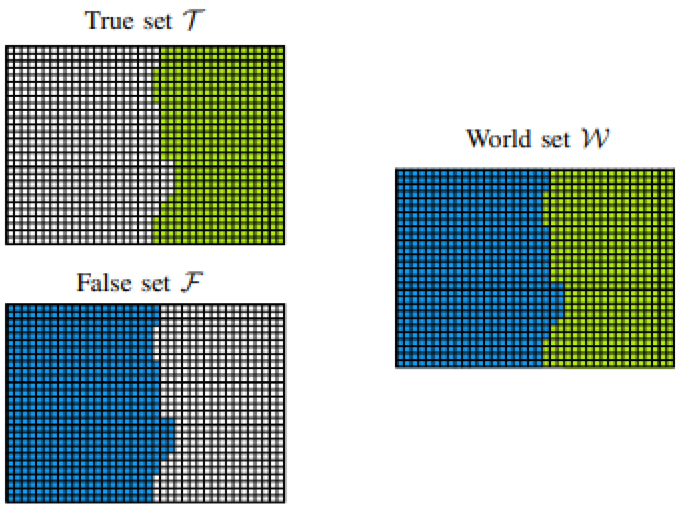

Figure 6: In probabilistic logic, there is no overlap between the sets of true T and false F propositions. Any mathematical object that does not belong to either set is probabilistically 1 for some other set.

In certain experiments, the variable z may fall under the intersection of T and F, resulting in the experiment being counted twice in both M1 and M2. The membership degrees of an element to the fuzzy sets F and T, respectively, are the normalized values µ1 and µ2, which correspond to the real values M1 and M2.

In general, whereas the sum of the degrees of membership $\sum_{i=1}^{Sc} = \pi i$ of a to se fuzzy sets that make up W may be smaller than, equal to, or bigger than 1, the sum of the probabilities $\sum_{i=1}^{Sc} = \mu i$ of a belonging to Sc crisp sets that build up the potential world W of the event E is necessarily 1.

Theory of Fuzzy Sets

Zadeh and Goguen authored the initial publications on fuzzy set theory in 1965, 1967, and 1969. The papers demonstrate the authors' intention to broaden the conventional concept of a set and a proposition by incorporating the notion of fuzziness as described.

In his 1965 publication on page 339, Zadeh discussed the concept of a fuzzy set, which allows for the construction of a conceptual framework that is comparable to conventional sets in several aspects. However, it is more versatile and may be applied to a broader range of applications, particularly in the fields of pattern classification and information processing. Essentially, this framework is a suitable approach for addressing problems in which the uncertainty stems from the absence of well-defined criteria for class membership, rather than random variables.

In this case, it might be helpful to list the main goals of this technology and say a few words about them. This is to correct the common misconception that fuzzy set theory or fuzzy technology is only or mostly useful for modeling uncertainty.

a) Modeling the uncertain

This aim is both the most popular and the oldest. However, I am uncertain if it can still be regarded as the primary objective of fuzzy set theory. Uncertainty has been a significant topic since the Middle Ages. Numerous ideas and methodologies purport to be the sole correct approach for modeling uncertainty. Typically, however, they either provide an inadequate definition of the term "uncertainty" or only do so in a limited manner. In my opinion, uncertainty, when viewed as a subjective concept, can and should be represented by several theories. The choice of theory depends on factors such as the underlying reasons of doubt, the types and quantity of available information, and the preferences of the observer. Fuzzy set theory is a versatile framework that can be employed to represent many forms of uncertainty in diverse scenarios. It could potentially rival other theories, although it may also be the most effective approach to represent this phenomenon in clearly defined circumstances. Providing an extensive discussion of this subject within the scope of this article [Zimmermann 1997] would be excessively lengthy.

b) Relaxing

Classical models and approaches frequently rely on dual logic as their foundation. They possess the ability to determine the feasibility, categorization, optimality, and suboptimality of various entities or scenarios. Frequently, this perspective fails to accurately depict the actual state of affairs. Fuzzy set theory has been extensively employed to transform the binary nature of conventional methods into a more gradual form. Fuzzy mathematical programming, fuzzy clustering, fuzzy Petri Nets, and fuzzy multi criteria analysis are all instances of this phenomenon [Zimmermann 1996; Bezdek and Pal 1992; Lipp et al. 1989; Zimmermann 1986].

c) Compactification

Due to limitations in both human and technological short-term memory, it is often not feasible to save all relevant data or present a large amount of information to someone in a way that makes sense to them. By leveraging linguistic variables or performing fuzzy data analysis, such as fuzzy clustering, fuzzy technology can reduce data complexity to a reasonable degree.

d) An easy way to find approximate solutions

During the 1970s, Prof. Zadeh expressed his intention for fuzzy set theory to be employed as a means of obtaining approximate solutions to real-world issues in a rapid and cost-effective manner. This objective has yet to be achieved in a satisfactory manner. However, in recent years, there have been notable instances that exemplify this objective. Bardossy [1996] demonstrated that fuzzy rule-based systems are more effective than systems of differential equations in tackling challenges related to modeling water flow. Upon comparing the outcomes of these two distinct approaches, it became evident that, in terms of practicality, the accuracy of the results was nearly same. This is particularly accurate when taking into account the errors and uncertainties present in the input data.

Fuzzy sets

A fuzzy set is a collection of objects that have varying degrees of membership, forming a continuum. This concept serves as the basis for fuzzy logic. A fuzzy set is a modified version of the conventional set. Elements of a fuzzy set exhibit a certain degree of membership with each other. The collection of aesthetically pleasing females serves as an illustration of a fuzzy set. The range of this gradation might vary from 0 to 1. Classical logic recognizes two distinct levels of membership: 0 for non-membership and 1 for full membership. The imprecise aspect of a set occurs due to the lack of well-defined bounds.

The system endeavors to emulate human cognition by regarding its components as variables inside a linguistic framework. Linguistic variables are represented by sentences, not numbers. The value of a linguistic variable is a sequence of discrete words. Language values can be categorized into many kinds. Two examples of linguistic elements are primary words, which serve as labels for certain fuzzy sets, and hedges, such as the term "very" when used to modify an atomic value. Fuzzy sets can be understood as qualities, whereas fuzzy logic serves as a method to draw conclusions in situations when data is inadequate or inconsistent. Linguistic descriptors such as "fast," "slow," "small," "large," "heavy," "low," "medium," "high,"

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"tall," etc. are denoted by fuzzy sets. An object has the ability to simultaneously be a member of many fuzzy sets.

Triangular, trapezoidal, extended bell-shaped, Gaussian, polynomial, and sigmoid functions are all types of fuzzy membership functions. The triangle, characterized by its three vertices, is the most basic and frequently encountered among these geometric figures. The membership is maximal at the center and minimal at the two extremes. Conversely, the trapezoid member function provides more comprehensive information. It can be described as a triangle curve that has been shortened, with a flat top. It resembles a quadrilateral rather than a triangle due to its four corners instead of three. The membership value of this interval is 1.

Three distinctive characteristics define fuzzy logic. Firstly, let us discuss the process of modifying language. The second distinctive feature is the depiction of relationships between variables using conditional expressions. Ultimately, fuzzy algorithms are utilized to analyze intricate relationships.

A construction industry "ecosystem" consists of a wide variety of individuals. A non-member element is denoted by the set 0,1> in conventional logic, while a member element is defined as one that does, and only one that does, constitute a set. Fuzzy logic, in contrast, expands this set to encompass the interval [0,1]. According to Zadeh (1992), fuzzy logic can be seen as a natural progression from more conventional systems. A construction industry "ecosystem" consists of a wide variety of individuals. A much of human thinking, particularly common sense reasoning, is inherently approximate, which is why it's important. Keep in mind that membership degrees are a very promising tool for quantifying qualitative (fuzzy) concepts. A fuzzy set is defined formally as follows:

Definition 1: A **Fuzzy Set**: A over a universe of discourse X (a finite or infinite interval within which the fuzzy set can take a value) is a set of pairs

$$A = \{\mu_A(x)/x : x \in X, \mu_A(x) \in [0,1] \in \Re\} \quad (1)$$

in where μA (X) is referred to as the degree to which element x belongs to the fuzzy set A. On the domain of real numbers, this degree falls somewhere between zero and one. When the value of μA(X) is 0, it means that x does not belong to the fuzzy set A at all, and when it is 1, it means that x belongs to the fuzzy set A fully. Keep in mind that the maximum uncertainty is at μA (X) = 0.5. An alternative to provide a complete inventory of all the hidden value pairs that comprise the set is to define the function μA (X), often known as the characteristic function or membership function.

The universe X may be called underlying universe or underlying domain and in a more generic way, a fuzzy set A can be considered as a function μ_A that matches each element of the universe of discourse X with its membership degree to the set A:

$$\mu_A(x): \quad X \rightarrow [0,1] \qquad (2)$$

The universe of discourse X or the set of considered values can be of these two types:

The intuitive definition of labels varies based on the context in which they are used as well as from person to person and moment to moment. A "high" person and a "high" building, for instance, do not measure the same.

Example: A linguistic variable is "Temperature." Using the membership functions shown in Figure 7, we may define four linguistic labels, such as "Very Cold," "Cold," "Hot," and "Very Hot."

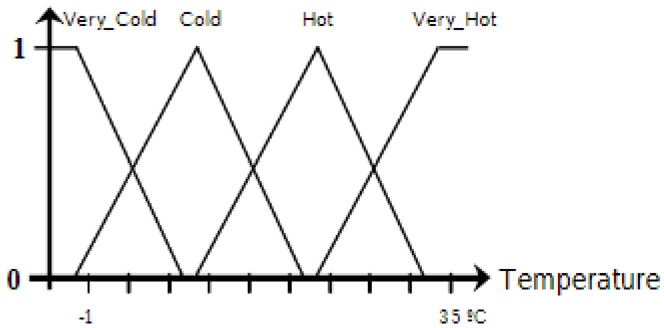

Figure 7: A Frame of Cognition with Four Linguistic Labels for Temperature (Example 1)

Characteristics and Applications

A thorough mathematical and computational theory has been developed from this fundamental idea, which has proved successful in tackling several problems (for references, see the citations at the beginning of the chapter). Multiple domains have made use of fuzzy logic: control systems, modeling, simulation, optimization, pattern recognition (including word recognition), information or knowledge systems (databases, knowledge management systems, expert systems, computer vision), biomedicine, picture processing, artificial intelligence, artificial life, and many more.

To summarize, fuzzy logic can serve as a valuable tool in situations where other approaches have proven ineffective in the past. It particularly excels in capturing intricate processes, necessitating the incorporation of expertise from experienced individuals, or when dealing with uncertain or difficult-to-measure magnitudes. Fuzzy logic is commonly employed when there is a requirement to represent and manipulate data that is uncertain, indistinct, or based on personal judgment.

Various applications integrate fuzzy logic with other general or soft computing technologies, such as rule-based systems, evolutionary algorithms, and neural networks.

Membership Functions

The membership degrees of fuzzy sets can be interpreted as conditional probabilities, as suggested in the initial investigation that explored the connection between fuzzy logic and probability (Loginov 1966). Subsequently, numerous individuals began to concur with this or a correlated perspective. Several prominent scholars, like Cheeseman (1988a, b) and Lindley (1987), contend that probability-based approaches, namely Bayesian methods, are sufficient for comprehending and handling uncertainty. As an example, let's examine this passage taken from (Lindley 1987):

"Probability is the sole adequate representation of uncertainty." What I am referring to is that every statement about uncertainty must be expressed as a probability. Additionally, several uncertainties should be integrated using the principles of probability, and the mathematical field of probability theory is sufficient to address any scenarios involving uncertainty. We refer to the concept of "the inevitability of probability."

Fuzzy logic explores a type of uncertainty that is fundamentally different from the uncertainty studied in probability theory, as previously noted by scholars such as Klir (1989) and Kosko (1990). An instance of this can be seen in the assertion "Peter is a tall individual." Fuzzy logicians consider this as a statement with multiple possible truth values, ranging from 0 to 1 (or another appropriate scale). Increased levels of precision correspond to higher levels of certainty. Human ideas, such as the notion of a tall man, have a comparable hierarchical organization, as do these claims. Several investigations conducted in the field of psychology of concepts (Belohlavek and Klir, 2011) have demonstrated the continuous and gradual character of human understanding. The phrase "Peter is a tall man." cannot be definitively assessed as a binary (yes/no) proposition. For example, the question "Is the proposition true, but answer 'yes' or 'no' only?" is inappropriate because it inaccurately depicts the ambiguity of the concept of a tall man. When probability theorists refer to a proposition's truth degree as a conditional probability, they are implying that the propositions are supposed to have two possible truth values, and that the truth degree quantifies the subjective uncertainty regarding the proposition's truthfulness. This perspective clearly diverges from that of fuzzy logicians. Fuzzy logicians criticize this viewpoint as fundamentally inadequate because of its ambiguous handling of fuzzy propositions.

1. (Figure 8) **Triangular**: defined by its modal value m, lower and upper bounds a and b, and upper bound b, such that amb. When it equals the value m-a, the value is referred to as the b-m margin.

$$A(x) = \begin{cases} 0 & \text{if } x \le a \text{ or } x \ge b \\ (x-a)/(m-a) & \text{if } x \in (a, m] \\ (b-x)/(b-m) & \text{if } x \in (m, b) \end{cases} \quad (5)$$

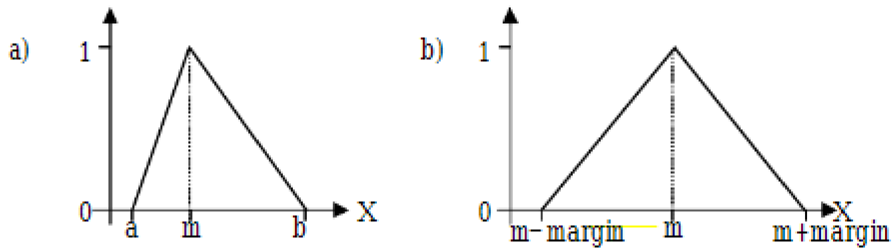

Figure 8: Triangular Fuzzy Sets: a) General. b) Symmetrical.

2. **Singleton** (Figure 9): It takes the value zero in all the universe of discourse except in the point m where it takes the value 1. It is the representation of a non-fuzzy (crisp) value.

$$SG(x) = \begin{cases} 0 & \text{if } x \neq m \\ 1 & \text{if } x = m \end{cases}$$

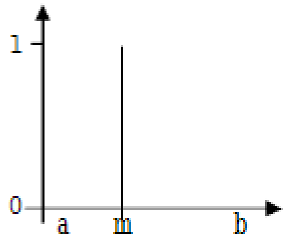

Figure 9: Singleton Fuzzy Set.

3. **L Function** (Figure 10): This function is defined by two parameters a, and b, in the following way, using linear shape:

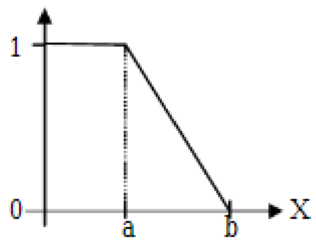

Figure 10: L Fuzzy Set.

4. **Gamma Function** (Figure 11): It is defined by its lower limit a, and the value k > 0. Two definitions:

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \leq a \\ 1 - e^{-k(x-a)^2} & \text{if } x > a \end{cases} \quad (8)$$

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \leq a \\ \dfrac{k(x-a)^2}{1+k(x-a)^2} & \text{if } x > a \end{cases} \quad (9)$$

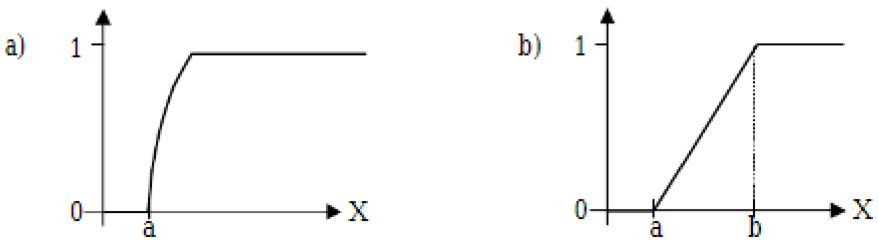

Figure 11: Gamma Fuzzy Sets. a) General. b) Linear.

Starting with a, this function exhibits fast expansion. The growth rate is proportional to the value of k. In the first definition, the growth rate is higher than in the second. Horizontal asymptote happening at 1.

- The gamma function is also expressed in a linear way (Figure 11 b):

$$\Gamma(x) = \begin{cases} 0 & \text{if } x \le a \\ \dfrac{x-a}{b-a} & \text{if } a < x < b \\ 1 & \text{if } x \ge b \end{cases} \qquad (10)$$

5. **S Function** (Figure 12): Defined by its lower limit a, its upper limit b, and the value m or point of inflection so that a<m<b. A typical value is: m = (a+b) / 2. Growth is slower when the distance a-b increases.

$$S(x) = \begin{cases} 0 & \text{if } x \le a \\ 2\{(x-a)/(b-a)\}^2 & \text{if } x \in (a,m] \\ 1-2\{(x-b)/(b-a)\}^2 & \text{if } x \in (m,b) \\ 1 & \text{if } x \ge b \end{cases} \qquad (12)$$

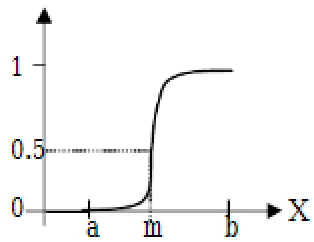

Figure 12: S Fuzzy Set.

Imprecision Without Fuzzy Logic

This section will present some principles that enable the handling of imprecise knowledge, without making reference to either fuzzy set theory or possibility theory. While several notions mentioned in this text have not yet been put into practice in any of the models, the models themselves are extensively examined in the bibliography section on imprecision in traditional databases. Codd (1979) initially introduced NULL values as a means to represent flawed data in databases, which were further expanded upon in following works (Codd, 1986, 1987, and 1990). The application of fuzzy set theory was not utilized in this model. A NULL value in an attribute indicates that the value can be any value inside the attribute's domain.

In Oracle SQL, a result that is neither True (T) nor False (F)—or unknown—is labeled "maybe" (m) if a comparison is made with a NULL value. For the classic comparators NOT, AND, and OR, the truth tables are shown in Table 1.

NOT	
T	F
m	m
F	T

AND	T	m	F
T	T	m	F
m	m	m	F
F	F	F	F

OR	T	m	F
T	T	T	T
M	T	m	m
F	T	m	F

Table 1: Truth Tables for the tri–valued logic: True, False and Maybe.

Later on, a more specific differentiation was established, categorizing the NULL value into two unique labels: the "A-mark," which denoted a missing or unknown value, despite being relevant, and the "I-mark," which indicated the lack of the value because it was not relevant or undefined. For instance, an individual who does not possess a car may display an I-mark on their license plate. This is a logic system with four possible values. By comparing any value that contains an A-mark, we may obtain the A value, which is similar to the m value in the three-valued logic mentioned earlier. The comparison of any value with an I-mark also results in the introduction of a new I value. The tetra-valued logic is shown in Table 2.

NOT	
T	F
A	A
I	I
F	T

AND	T	A	I	F
T	T	A	I	F
A	A	A	I	F
I	I	I	I	F
F	F	F	F	F

OR	T	A	I	F
T	T	T	T	T
A	T	A	A	A
I	T	A	I	F
F	T	A	F	F

Table 2: Truth Tables for the tetra–valued logic.

Prospects for the Future

The recent editions of fuzzy sets and systems contain numerous examples of such applications, including the effective resolution of systems of differential equations (see to [Bardossy 1996]). However, overall, fuzzy set theory has not yet proven its computational ability to properly solve large and intricate problems. The reason for this is that either conventional computing methods (such as linear programming, branch and bound, and traditional inference) are still necessary, or the inclusion of more information in fuzzy set models results in excessively intricate computations. In this case, the combination of prudent standards, which include support for fuzzy logic, and successful algorithmic mashes-ups of heuristics and fuzzy set theory, may demonstrate significant promise. Put simply, there is an urgent requirement for conducting research on fuzzy algorithms.

Since 1970, decision analysis has been a significant field where fuzzy set theory has been widely applied. Due to the extensive nature of this textbook, just a single chapter could be allocated to this particular subject. Additional information can be found in several publications and papers listed in the bibliography, including my book "Fuzzy Sets, Decision Making and Expert Systems" (1987, third edition 1993). Further research endeavors are expected to advance this field and aid in the closure of remaining gaps.

Conclusion and Results

An advantageous aspect of fuzzy set theory is its high level of generality. This implies that it has the capacity to effectively manage numerous emerging advancements required to address existing difficulties and future concerns. Several domains, such as possibility theory [Dubois and Prade, 1988a], fuzzy clustering, fuzzy control, and fuzzy mathematical programming, have already achieved significant advancements. Nevertheless, there are still ample opportunities for expansion in various other locations.

Fuzzy control is the primary domain in which fuzzy set theory is widely recognized and attracts significant attention from scientists, students, and practitioners. Noteworthy publications such as [Babuska 1988] and [Verbruggen et al. 1999] provide current insights into the progress of this field. Regrettably, the widespread appeal of this discipline has obscured the full potential of fuzzy set theory. Upon completion of this book, we anticipate that the reader will possess a comprehensive understanding of the several uncharted avenues for implementing this idea.

In order to address these issues, a substantial amount of research, encompassing both formal and empirical methods, will be necessary. The accomplishment of most of this research will rely solely on the collaborative efforts of interdisciplinary teams. Allow us to highlight certain areas of research that require attention. Fuzzy set theory serves as a modeling language for imprecise and intricate formal and factual structures. While various additional connectives, ideas, and operations have been proposed in the literature, the predominant utilization and application of fuzzy set theory has been focused on the min-max version. Membership functions are intended to be provided. In order to effectively utilize fuzzy set theory as a modeling language, extensive empirical study and meticulous modeling work on connectives and the measurement of membership functions are necessary. Within the realm of artificial intelligence, there exist untapped prospects that have yet to be capitalized upon. Most of the methodologies and approaches proposed thus far have been binary. In order for artificial intelligence to effectively capture human thinking and perception, the phenomenon of uncertainty must be described with greater accuracy than it has been thus far. Fuzzy set theory provides numerous potentials in this context.

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