



## Fuzzy Logic: A Deeper Dive into Fuzzy Functions

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## Abstract

In 1965, the idea of a "fuzzy set" was created. Since then, this key set has been used by scholars across numerous fields. To model human participation in human-based intelligence to achieve modernity in many fields, such as data analysis, data mining, image coding and explaining, and intelligence systems, fuzzy set has proven to be quite promising and effective. Because of its wide applicability and fruitful applications, fuzzy set theory is now a recognized area of study in both pure and practical branches of mathematics and statistics. Fuzzy sets have been a central area of study for quite some time, and yet they continue to pique the interest of scholars who hope to find answers to pressing problems by challenging fundamental assumptions. Through collaboration, the members of a fuzzy set can efficiently address a wide variety of physical world problems that may be outside the purview of traditional methods. This suggests that the fuzzy set may be adaptable enough to deal with a wide variety of issues, including, but not limited to, those involving decision making, intelligent data analysis, information processing, pattern recognition, and optimization.

The purpose of this Special Issue is to go more deeply into the recent developments in fuzzy set theory and the broadening of fuzzy set theory to include applications in other areas of mathematics and engineering, such as group theory, ring theory, statistics, topological spaces, graph theory, decision making, and other fields of applied science.

**Keywords:** Fuzzy Logic, Fuzzy functions, Fuzzy sets, membership, trends.



## Introduction

Knowledge-based systems are computer programs that attempt to mimic human intelligence in order to make decisions and offer recommendations. Knowledge-based systems include the illustrious Expert System. Despite its widespread use, such systems cannot model the ambiguities and vagueness of problems in the real world. Expert system researchers paid little attention to fuzzy logic after its introduction by Lotfi Zadeh in 1965. "The initial reception of the concept of a linguistic variable was far from positive," Zadeh said, "largely because my advocacy of the use of words in systems and decision analysis clashed with the deep-seated tradition of respect for numbers and disrespect for words." Fuzzy logic was developed with the intention of illuminating the hidden dimensions of traditional reasoning. The best method to simulate human thought is through this style of reasoning. Fuzzy logic has been around for forty years, but it is only now gaining traction in the field of artificial intelligence. It is being implemented in expert systems to deal with the fuzziness and uncertainty of real-world issues. A Fuzzy Expert System is an expert system that makes use of a set of fuzzy sets and rules to make reasoning easier. The following is the outline for this paper: Following a brief introduction to fuzzy logic, we will move on to discuss fuzzy expert systems in greater depth. Next, we will examine several examples of fuzzy expert systems. The findings would be the last part of this paper.

## What is fuzzy logic?

In contrast to the binary truth, which can only take the values 1 or 0, fuzzy logic is a method for problem resolution that suggests numerous truth variables.

Saying "Yes" or "No" isn't always an option in real life. This is because you can be put in a position where you don't have all the facts to make an informed decision. Or maybe you're



just a little confused. You probably won't immediately answer "Yes" or "No" to a question regarding your availability on a specific date next month because you don't know for sure what your plans are.

I know, it's not easy. Fuzzy logic is an AI technique that helps computers deal with uncertain input data like these.

A machine learning framework or artificial intelligence may employ fuzzy logic as a means of making decisions. Simply said, it's the process of comparing real-world variable values between 0 and 1. Between 0 and 1, real numbers are represented using fuzzy logic.

The term "fuzzy" is used to describe something that is unclear. It's possible that the computer won't be able to determine if a given situation is true or false. In Boolean logic, a value of 1 means "True" and a value of 0 means "False." A fuzzy logic approach, on the other hand, takes into account all the gray areas of an issue, where there may be more than just two possible answers (True and False).

The application of fuzzy logic allows for more accurate numerical answers to problems. Fuzzy logic considers human thought to be the most reliable and precise data type. A decision tree analysis's exact worth is captured here.

Lotfi Zadeh, a professor at the University of California at Berkeley, developed fuzzy logic in the 1960s. He felt that traditional computer logic was incapable of dealing with ambiguous or inaccurate information. A computer, like a human, may consider a variety of alternatives to True and False. Such things may Yes, Possibly Yes, Unsure, Perhaps No, and No Way!

One method for controlling the actions of a physical system is the fuzzy logic controller (FLC). FLC controls a wide variety of systems, including the washer, air conditioner, heater, fan regulator, traffic controls, and brake controller. FLC is used when a mechanistic mathematical explanation of the problem cannot be obtained.

Fuzzy logic, which is based on the way humans make decisions, is useful for modeling complex problems with imprecise or incomplete information. Because of their familiarity with everyday language, fuzzy logic programs are more easily implemented than their logical and object-oriented counterparts. And because fewer instructions are required, less memory is required for its operation.

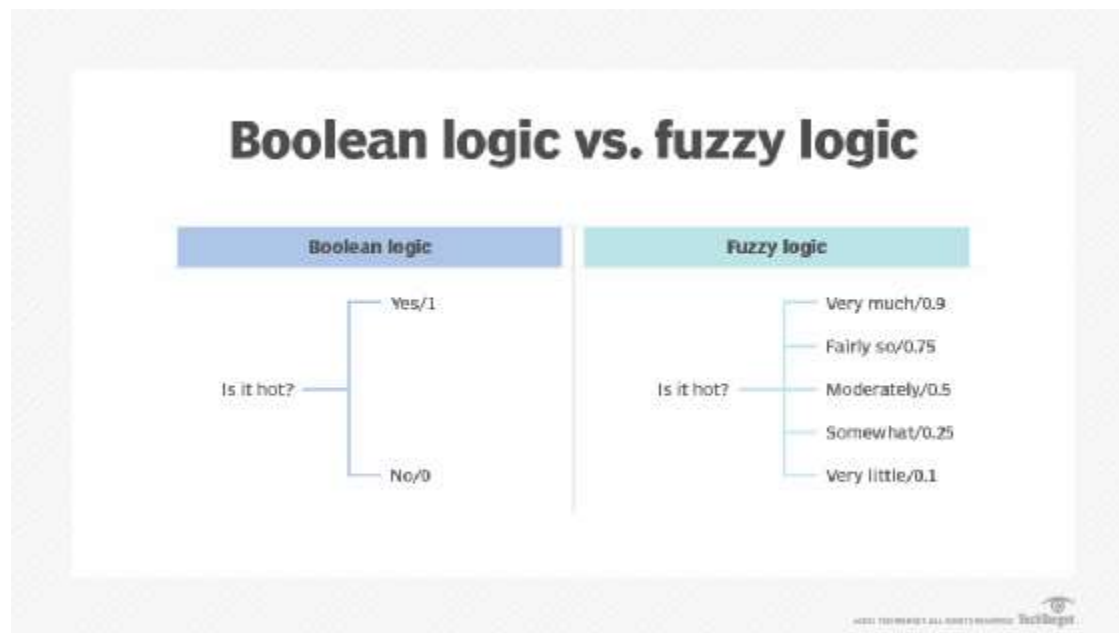


Figure 1: Comparing the degrees of truth with Boolean logic vs. fuzzy logic



## Mechanism of Fuzzy Logic

Fuzzy logic has several uses for modeling human cognition in artificial settings. It is more concerned with plausible reasoning, which is more in line with how the real world works. It is built to process ambiguity and is good at extrapolating results. The stages to using fuzzy logic are as follows:

Determine what it is you want to manage, the appropriate response, any potential system problems, and how to use the system.

Establish the connection between the input, the output, any variations or errors, and the exceptions: For fuzzy logic to work, users must first choose which inputs are most important while also selecting an appropriate error rate and a minimal amount of input variables.

The first step in developing an if/then rule is to break down the control problem into a series of "IF A AND B THEN C" rules using the rule-based architecture of fuzzy logic. These rules will define the desired action for given inputs. The complexity of a rule is proportional to the number of inputs and the number of variables associated with those inputs.

Function code, please write: Design a membership function for fuzzy logic that specifies the relative importance of input and output data.

Walk us through a typical business cycle: Develop a process for fuzzy logic before and after implementing it in hardware or software.



Prove your setups: You can fine-tune membership functions and rules through system testing and outcome evaluation. Maintain a cycle of experimentation and fine-tuning until you reach your goals.

### Significance of Fuzzy Logic

Fuzzy logic's foundational form is built using decision tree analysis. As a result, it provides the basis for AI algorithms that draw judgments based on rules. The programming of AI systems relies heavily on fuzzy logic and fuzzy semantics (linguistic variables). The number of businesses using AI solutions and applications keeps growing, even as fuzzy logic's programming capabilities advance.

IBM's Watson is one of the most well-known AI systems out there, and it makes use of a technique called "fuzzy logic" or "fuzzy semantics." Fuzzy logic, in conjunction with machine learning and associated technological systems, is used to provide investment intelligence reports in the financial services sector.

There is often a lumping together of machine learning with fuzzy logic. But they're not the same thing either. The term "machine learning" is commonly used to refer to computing systems that mimic human intellect by modifying algorithms to regularly solve complex problems. Fuzzy logic is a set of rules or functions that can be applied to imperfect data, but the underlying processes still require human input. Both can be used to solve difficult issues and advance artificial intelligence.

Data mining is the process of searching for and extracting useful information from large data collections. It lies at the crossroads of the fields of mathematics, computer science, and machine learning. The principles of fuzzy logic can be applied to fuzzy data sets to



arrive at reasonable conclusions. Fuzzy logic is useful for finding significant correlations in this data since data mining frequently entails imprecise measurements.

Some of the most advanced trading algorithms make use of fuzzy logic mathematics to automatically produce buy and sell signals for analysts. These innovations let financiers react to a wide range of market conditions that can have an effect on their holdings.

Washing machines and dryers also make extensive use of fuzzy logic. Sensors in washing machines powered by fuzzy logic constantly track the water's temperature. It adjusts the mechanisms and procedures accordingly. Using these techniques will save you time and money.

In order to get the best possible cleaning outcomes, fuzzy logic controls variables such as water temperature, spin speed, wash time, amount of water used, and effectiveness of the final rinse. They recall washing algorithms from before and use that knowledge to optimize the washing process in the future.

Laundry systems today are typically constructed with energy efficiency in mind. These technologies help you save energy even if you wash a lot of clothes twice or three times a week. The sensors track the status of the whole washing cycle to make any necessary adjustments for optimal cleaning.

Companies like Panasonic use comparable technology in their dishwashers. Dishwashers' wash cycles and cleaning methods can be adjusted with the use of fuzzy logic. The dishwasher's performance declines as the number of dishes increases.



## The Pros of Using a Fuzzy Logic System

This system is designed to accept inaccurate, skewed, or noisy data from any source.

Fuzzy Logic Systems are simple and straightforward to build.

The explanation behind the inclusion of set-theoretic mathematical notions in fuzzy logic is straightforward.

Since it is similar to human reasoning and decision-making, it is able to effectively solve difficult problems across many disciplines.

The algorithms are easily stated with minimal information, necessitating little storage space.

## Fuzzy Logic Systems' Drawbacks

Many researchers have offered conflicting solutions to the same problem using fuzzy logic, leading to confusion. Fuzzy logic does not provide a methodical means of resolving any situation.

The lack of a mathematical explanation of our method makes it hard, if not impossible, to prove its properties.

Since fuzzy logic can use both exact and approximate information, it is often inaccurate.

## Fuzzy Sets

"A fuzzy set is a class of objects with a continuum grades of membership," which is the foundation of fuzzy logic. A fuzzy set is a variant on the standard set. Elements of a fuzzy set share some level of membership with one another. The set of attractive women is an



example of a fuzzy set. This gradation might be anything from 0 to 1. In classical logic, there are only two possible membership levels: 0 for non-membership and 1 for full membership. The fuzzy nature of a set arises because its boundaries are not well defined.

It attempts to mimic human thought by treating its pieces as variables in a language. Sentences, rather than numbers, are the values of linguistic variables. A linguistic variable's value is a string of individual words. Values in language can be broken down into various classes. Primary words, which are labels for certain fuzzy sets, and hedges, such as very with regards to the atomic value, are two examples.

So, fuzzy sets are really just attributes, and fuzzy logic is just a means to reach conclusions when you have incomplete or conflicting data. Common linguistic descriptors such as "fast," "slow," "small," "large," "heavy," "low," "medium," "high," "tall," etc. are represented by fuzzy sets. An item can belong to multiple fuzzy sets at once.

Triangular membership functions, trapezoidal membership functions, extended bell-shaped membership functions, Gaussian curves, polynomial curves, and sigmoid functions are all examples of fuzzy membership functions. The triangle, with its three points, is the simplest and most common of these shapes. The membership is one at the peak and zero at the two ends. The information provided by the trapezoid member function, on the other hand, is more extensive. It's essentially a truncated triangle curve, with a level top. It's like a triangle, except with four corners instead of three. The membership value of this interval is 1.

There are three hallmarks of fuzzy logic. To begin, let's talk about how language can be changed. The second distinguishing characteristic is the representation of relations between



variables through conditional expressions. Finally, fuzzy algorithms are applied to complex relations.

### A Comparison between Fuzzy Sets and Classical Sets

Classical sets (or just 'sets') are something that everyone is probably extremely comfortable with. An element can only belong to or not belong to a classical set. Take the set "1,2,0" as an illustration. The number one is unquestionably included in the set. The number 10...what's up with that? We can also state categorically that it is not a member of this group. As you can see, then, there is a fairly well-defined basis behind inclusion in a canonical or element set. The "membership" of an element is defined as 1 if and only if it is included in the classical set. The "membership" of an element is defined as "zero" if it does not belong to the classical set.

In mathematical notation, the "membership" of an element can be represented by a function known as the membership function. Simply said, there are just two possible values for a traditional set's membership function: 1 and 0.

Why not use a fuzzy set instead? Things aren't quite what they seem to be there. (So, does that mean the end of fuzzy logic? Listen to me out before you jump to that conclusion!

One definition of a fuzzy set is a set in which the membership of individual components cannot be determined with absolute certainty. An item may only partially belong to the set. (This is the meaning of "partial truth" on Wikipedia. Yes, I did say that I will address it again. Therefore, in fuzzy logic, the membership function is not a binary set of 1 or 0.

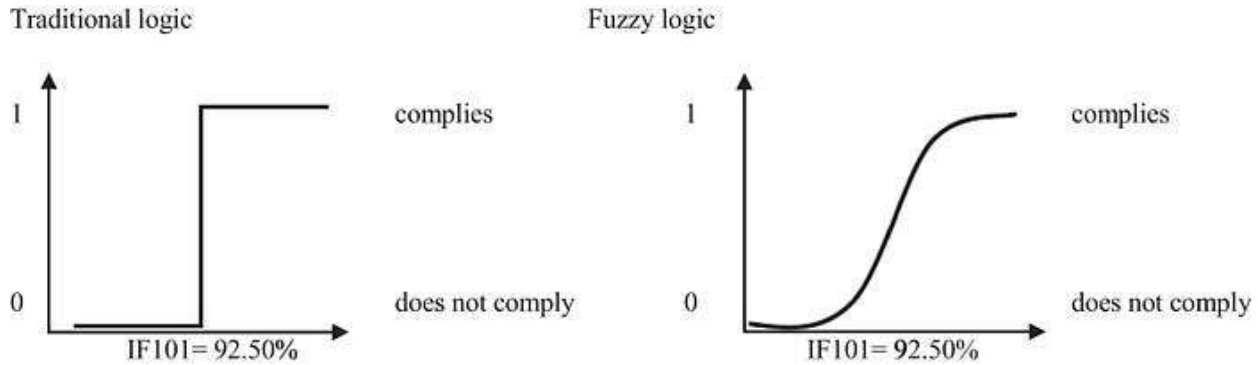


Figure 2: Diagram juxtaposing the difference between traditional(classical) logic and fuzzy logic. (Pic courtesy: ResearchGate)

### Difference Between Crisp Set and Fuzzy Set

The objects in a crisp set share the traits of being countable and finite. The set 'X' is a finite set if and only if it contains exactly the same subset of items as the universal set, U. Since it is unclear if the arbitrary member is part of X or not, we have two options for defining the set. These are the first elements that would be added to set X, unless they are not part of X.

One definition of a fuzzy set is "the integration of elements with varying degrees of membership in the set." The term "fuzzy" suggests ambiguity, but we can also say that the replacement among different membership degrees emphasizes the fuzzy set's fuzziness and ambiguity. Therefore, the universe's elements are measured against a function designed to detect uncertainty and ambiguity to determine how many elements belong to the set.

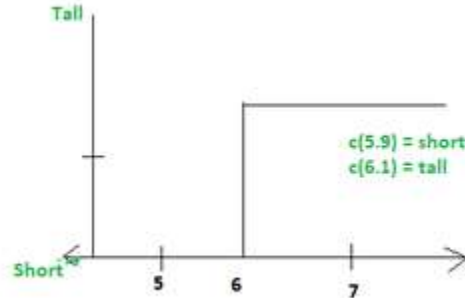


Figure 3: Crisp Set

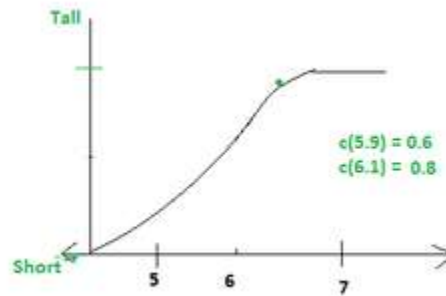


Figure 4: Fuzzy Set

### What is Fuzzy Function?

In the field of fuzzy theory, the term "fuzzy function" can be understood in at least two distinct ways. Fuzzy functions, theoretically speaking, are subclasses of fuzzy relations that share the generalized uniqueness property. In this method, a degree of importance is assigned to each element in the domain of a fuzzy function. Therefore, degrees must be used instead of straight functional values.



In his early work, L. Zadeh presented the now-famous extension concept as an alternative, more pragmatic way of looking at fuzzy functions. This concept states that any function (in the traditional sense) can be "fuzzified," or generalized to include arguments in the form of fuzzy sets. In this way, a mapping between a set of fuzzy subsets of the domain and a set of fuzzy subsets of the range can be determined for any ordinary function. Fuzzy functions are defined as ordinary mappings between two universes of fuzzy sets, a technique we adopted in [9]. In [10], and indirectly in many other publications devoted to fuzzy IF-THEN rules models, where these models are employed as partially supplied fuzzy functions, a definition quite similar to this one was adopted.

### Preliminaries

Fuzzy set theory and its applications have benefited greatly from an extension principle first stated by L. Zadeh in 1975. Let's review the guiding concept and propose the relational form that will be used to fuzzy function in the future.

Assume  $f: X \rightarrow Y$  is a function, and that  $X$  and  $Y$  are universal sets. Moreover, let  $F(X)$ ,  $F(Y)$  be the domains of fuzzy sets that are uniquely identified by their membership functions, i.e.  $F(X) = A: X [0, 1]$  and similarly for  $F(Y)$ . For all  $A \in F(X)$ , there exists  $f: F(X) \rightarrow F(Y)$  that satisfies the extension principle.

$$f(A)(y) = \sup_{y=f(x)} A(x). \quad (1)$$

Let  $R_f$  be a binary relation on  $X \times Y$  which corresponds to the function  $f$ , i.e.

$$R_f(x, y) = 1 \Leftrightarrow y = f(x).$$



Then it is easy to see that (1) can be equivalently represented by

$$\hat{f}(A)(y) = \sup_{x \in X} \min(A(x), R_f(x, y)). \quad (2)$$

The relational form of the extension principle is expression (2). If fuzzy set A is L-valued (see Definition 2 below), if binary relation R<sub>f</sub> is a fuzzy relation, and if min is substituted by a monoidal operation, then the meaning of equation (2) will be more general.

### Membership Function of Fuzzy Logic

A mapping between each point in the input space and a membership value between 0 and 1 is defined by this graph. The input space, also known as the universe of discourse or universal set (u), includes everything that could be relevant to a given application.

Most fuzz effects fall into one of three categories:

Single-Input Multiple-Output Fuzzer

A Gaussian fuzzy-maker

Fuzzcutters of trapezoidal or triangular shape

### A definition of Fuzzy Control.

It's a method for programming AI-like behavior into a machine's control infrastructure.

While it may not be built to provide perfect logic, it should be able to provide reasonable explanations.

It can reason like a person does, drawing inferences based on what it already knows.

Fuzzy logic makes it simple to cope with ambiguities.

The membership function in fuzzy logic is continuous between 0 and 1, meaning that it can take on any real value between those two extremes. The former indicates that the element does not belong to the fuzzy set, whereas the latter indicates that it fully does. Any non-zero number indicates that the element is only partially included in the set.

In fuzzy logic, the membership function stands in for the level of veracity.

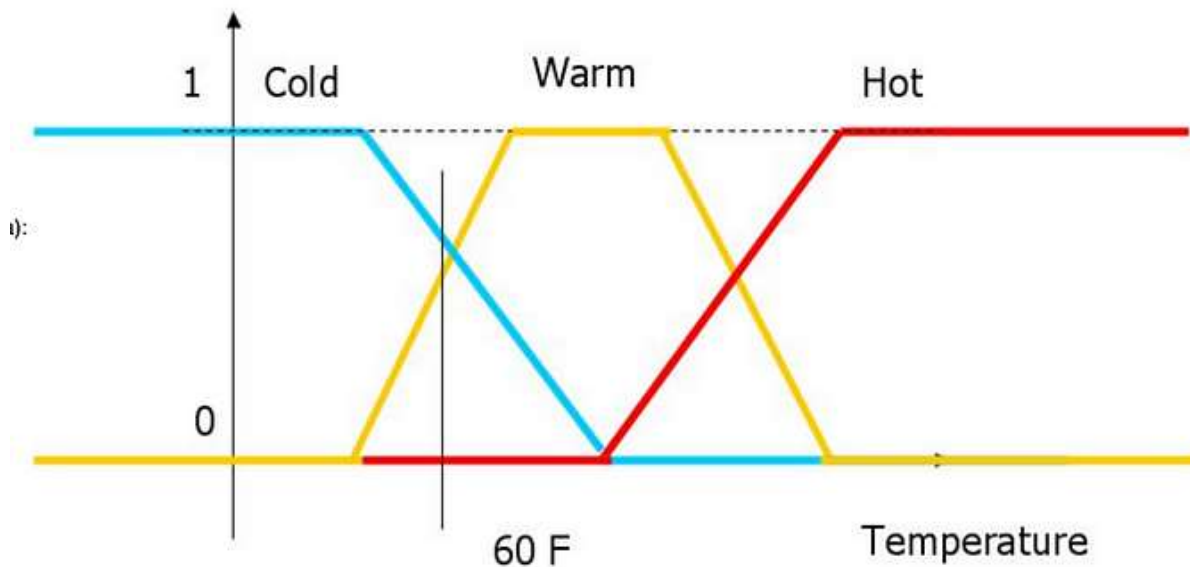


Figure 5: Diagram depicting membership function for temperature (Pic Courtesy: [cds.caltech.edu](http://cds.caltech.edu))

### Fuzzy Operations:

Let  $f_A(x)$  represent the quality of  $x$ 's membership in  $A$  to illustrate a fuzzy logic operation.

Fuzzy set  $A$  has a complement or negation defined as  $A'$  by the formula:



$$f_{A^c}(x) = 1 - f_A(x) \quad (1)$$

given a two fuzzy sets A and B, the notion of containment is defined by

$$A \subset B \leftrightarrow f_A \leq f_B \quad (2)$$

The maximum membership function of two fuzzy sets A and B is given by their union (equivalent to or), which may be written as

$$\text{Max}[f_A(x), f_B(x)] \quad (3)$$

It equals:

$$f_A \vee f_B \quad (4)$$

When two fuzzy sets A and B meet (represented by and), their membership functions are minimized to provide the set

$$\text{Min}[f_A(x), f_B(x)] \quad (5)$$

It equals:

$$f_A \wedge f_B \quad (6)$$

In addition to the aforementioned, fuzzy logic also includes operations for representing linguistic heuristics. The fuzzy subset of A that is the outcome of the concentration operation has smaller membership magnitudes than the original fuzzy set.

$$\text{CON}(A) = A^2 \quad (7).$$

Dilation is defined as an operation with the opposite effect of concentration procedures.

$$DIL(A) = A^{0.5} \quad (8)$$

When applied to a non-fuzzy set, the fuzzification operation makes the set's borders fuzzy, turning it into a fuzzy set A. The fuzziness of a fuzzy set can be increased with the use of the wavy bar, hence the name. This process is characterized by:

$$\int_v \mu_A(y) K(y) \quad (9)$$

Where  $\mu_A(y)$  is the degree of membership y have in A and  $K(y)$  is the fuzzy set resulting from applying the fuzzifer to  $1/y$

To compute the meaning of linguistic variables, hedges are regarded as operators. Let x be an atomic linguistic variable then very x is given by:

$$\text{very } x = x^2 \quad (10)$$



Figure 6: Venn diagrams for  $A \cap B$ (intersection),  $A \cup B$ (union).

### Fuzzy Systems:

The subway system in Sendai, Japan, is currently the most well-known fuzzy system in the world. They also apply fuzzy logic in a wide range of other applications. Robots, optical



fuzzy systems, and the ability to read handwriting and characters are just a few examples. Japan was an early adopter of fuzzy logic and is currently a global leader in the field of commercial fuzzy logic application development. Sony also makes an auto-adjusting, fuzzy TV that modifies the picture's brightness, contrast, and color for you. Fuzzy logic has also been used by well-known companies like Nissan and Mitsubishi. Fuzzy logic has also been employed in the creation of educational and weather prediction systems. Since geological entities have their own unique set of fuzziness, fuzzy logic finds use in many other disciplines as well.

#### Applications of Fuzzy Logic in Expert Systems:

Fuzzy logic has been widely used in the creation of expert systems. This has led to the development of fuzzy expert systems that can "think" and "reason" like a human being. We will highlight some of the many examples of fuzzy expert systems published in the literature.

Successful web-based fuzzy expert system for assisting novice managers in estimating the likelihood of the emergence of various problems and providing advice to prevent or mitigate the effect of such problems in landfill operations is described in [3]. It's a way for managers to rate things like the amount of mud on the road not with numbers but with words. Because there is no historical information on landfill components, no measurements are available, and the causes of landfill problems are unclear, landfill operations are often regarded uncertain.

Concerning issues with power quality, [4] details a fuzzy expert system. Because there are so many variables that might affect power quality, the system gathers data on both the physical location and the connected devices. Fuzzy facts are gathered from the user for



metering purposes, including current, frequency, and voltage. The system determines what's best to recommend based on the data it has collected. The FuzzyClips framework was used to create this system.

The hospitality and travel sectors have not paid much attention to fuzzy logic. In [5], we read about a hotel-selection expert system that uses fuzzy logic. The system's goal is to facilitate the search for hotels that meet the needs of individual travelers. They can use linguistic values for many variables to communicate their choices. Hotel rates, for instance, might range from very affordable to rather pricey. Visual Basic 6.0 has been used in the system's development.

In [6], an example of a fuzzy expert system for water supply forecasting is presented and discussed. The requirement for a system whose efficiency is not affected by the quantity of available historical data drove the design of this system. The MatLab fuzzy logic toolbox was used in the development of this system. The method took into account the knowledge of experts as well as data from the previous three years. It was determined that the created technique outperformed more conventional approaches to water supply forecasting.

Software for Weather Analysis The meteorological data we have are imprecise. There is no clear definition of "weather information." This justifies the use of fuzzy expert systems in the study of weather prediction. SIGMAR, a fuzzy expert system, is presented in [8] to aid in weather monitoring. The system's goal is to keep tabs on wind speeds and sound an alarm if they rise above the predicted levels. The technique has been found to be very helpful in the winter by weather forecasters. The economic toll of ice jam floods in Alberta, Canada, is in the millions. Several unknown causes combine to cause floods due to ice



jams. Therefore, a method for predicting ice jams is required. [7] An investigation of the feasibility of using fuzzy expert systems for ice jam risk prediction. The preliminary system's results are encouraging, but further study is required.

## Conclusion

Fuzzy Logic is a form of AI developed to deal with fuzziness and imprecision. Fuzzy logic provides for the handling of incomplete or ambiguous data and the representation of fuzzy ideas. From decision-making and pattern identification to control systems and robotics, Fuzzy Logic has many real-world uses. Fuzzy Logic can be used in climate control systems, for instance, to adjust the indoor environment in response to inputs such as room size, occupancy, and weather. Fuzzy sets, which can have several, partially matching members, are the foundation of Fuzzy Logic.

In contrast to the yes/no nature of standard binary sets, membership in a fuzzy set can be more nuanced. A person's height, for instance, could be represented by a fuzzy set that includes the extremes of "short" and "tall," as well as intermediate values. Fuzzy Logic excels in real-world situations where data is often partial or unclear because of its ability to deal with such data. Combining Fuzzy Logic with other AI methods, such as Neural Networks, can result in even more advanced models. However, Fuzzy Logic needs careful selection of membership functions and fuzzy rules, which can be computationally expensive. Another potential problem of Fuzzy Logic is that it can be hard to understand and articulate.



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